The Priory Academy LSST

Mathematics A-Level

GCSE to A-Level transition

At The Priory Academy LSST the Mathematics and Further Mathematics A level courses both follow the OCR-MEI specification.

The Transition from GCSE to A level is hard! However, it is much easier if you have a very thorough understanding of your GCSE work and are able to reproduce that work at the beginning of the A level course.

The A level course covers Core Mathematics, Statistics and Mechanics, but all of these require you to be able to manipulate number and algebra accurately. The following questions are all on topics that you should be comfortable with from Higher GCSE and we would expect that any student intending to be successful at A level is able to complete the majority of the following work successfully.

Surds

Write these in terms of the simplest possible surd.

(i)
$$\sqrt{8}$$

(iv)
$$\sqrt{216}$$

(v)
$$\sqrt{63}$$

(vi)
$$\sqrt{300}$$

Simplify the following

(i)
$$(1+\sqrt{2})+(3-2\sqrt{2})$$

(i)
$$(1+\sqrt{2})+(3-2\sqrt{2})$$
 (ii) $(5\sqrt{2}-2\sqrt{3})-(\sqrt{2}+3\sqrt{2})$

(iii)
$$2(\sqrt{5}-3\sqrt{3})+3(2\sqrt{5}+\sqrt{3})$$
 (iv) $\sqrt{18}+\sqrt{72}-\sqrt{98}$

(iv)
$$\sqrt{18} + \sqrt{72} - \sqrt{98}$$

Multiply out the brackets and simplify as far as possible.

(i)
$$(1+\sqrt{2})(3-\sqrt{2})$$

(ii)
$$(2-\sqrt{3})(3+2\sqrt{3})$$

(iii)
$$(3-2\sqrt{5})(1-3\sqrt{5})$$

(iv)
$$\left(3-\sqrt{2}\right)^2$$

Rationalise the denominators of the following.

(i)
$$\frac{3}{\sqrt{3}}$$

(ii)
$$\frac{1}{\sqrt{5}}$$

(iii)
$$\frac{1+\sqrt{2}}{\sqrt{2}}$$

(iv)
$$\frac{1}{\sqrt{3}+1}$$

(v)
$$\frac{\sqrt{2}}{2-\sqrt{2}}$$

Extension

Express each of the following expressions as a single rational fraction, leaving a rational denominator.

(i)
$$\frac{2}{\sqrt{7}} + \frac{3}{\sqrt{2}}$$

(ii)
$$\frac{1}{3-\sqrt{2}} + \frac{2}{2-\sqrt{3}}$$

(iii)
$$\frac{3}{\sqrt{x}} - \frac{\sqrt{x}}{4}$$

(iv)
$$\frac{1}{x + \sqrt{y}} + \frac{1}{x - \sqrt{y}}$$

Indices

Do not use a calculator in this exercise.

Find:

(ix)
$$36^{-3/2}$$

(x)
$$\left(\frac{1}{2}\right)^{-1}$$

(ii)
$$2^{6}$$
 (iii) $4^{1/2}$
(v) 5^{-2} (vi) $64^{1/3}$
(viii) $8^{5/3}$ (ix) $36^{-3/2}$
(xi) $\left(\frac{25}{9}\right)^{-1/2}$ (xii) $\left(\frac{27}{64}\right)^{-2/3}$

(xii)
$$\left(\frac{27}{64}\right)^{-2/3}$$

2. Simplify the following:

(i)
$$3^{11} \times 3^{-4} \div 3^{3}$$

(ii)
$$(2^5)^3 \times (2^7)^{-2}$$
 (iii) $\frac{5^6}{5^5 \times 5^3}$

(iii)
$$\frac{5^6}{5^5 \times 5^3}$$

Simplify:

(i)
$$2^3 \times 16^{\frac{1}{2}}$$

(ii)
$$\frac{3^5 \times 5^3}{\sqrt{81 \times 25}}$$

Extension

Simplify:

(i)
$$\frac{2^{\frac{5}{2}} - 2^{\frac{3}{2}}}{2^{\frac{1}{2}}}$$

(ii)
$$\left(\frac{x^{\frac{7}{4}} - x^{\frac{3}{4}} + x \times x^{\frac{7}{4}}}{x^{\frac{1}{4}}}\right)^2$$

(iii)
$$\left[\frac{y^{\frac{1}{2}}}{x^{\frac{3}{4}}} - \frac{x^{\frac{5}{4}}}{y^{\frac{3}{2}}} \right]^4$$

Quadratics

Factorise these quadratic expressions.

(i)
$$x^2 + 5x + 6$$

(ii)
$$x^2 + x - 12$$

(iii)
$$x^2-9$$

(iv)
$$x^2 - 6x + 8$$

$$x^{2} + 5x + 6$$
 (ii) $x^{2} + x - 12$
 $x^{2} - 6x + 8$ (v) $2x^{2} + 3x + 1$

(vi)
$$3x^2 + x - 2$$

(vii)
$$4x^2 - 8x + 3$$
 (viii) $4x^2 - 25$

(viii)
$$4x^2 - 25$$

(ix)
$$6x^2 - x - 12$$

Factorise:

(i)
$$x^2 - 4x$$

(ii)
$$x^2 - 17x - 60$$

(iii)
$$x^2 + 4(x+1)$$

(iv)
$$3x^2 - 11x + 6$$

Solve these quadratic equations by factorising.

(i)
$$x^2 + 4x + 3 = 0$$

(ii)
$$x^2 + 5x - 6 = 0$$

(iii)
$$x^2 - 6x + 8 = 0$$

(iii)
$$x^2 - 6x + 8 = 0$$
 (iv) $x^2 - 7x - 18 = 0$

(v)
$$2x^2 + 5x + 3 = 0$$

(vi)
$$2x^2 + x - 6 = 0$$

4. Write down the equation of the line of symmetry and the coordinates of the vertex of each of the following quadratic graphs:

(i)
$$y = (x-4)^2 + 1$$

(ii)
$$y = (x+2)^2 - 3$$

(i)
$$y = (x-4)^2 + 1$$
 (ii) $y = (x+2)^2 - 3$
(iii) $y = (2x-1)^2 - 5$ (iv) $y = 3 - (x+1)^2$

(iv)
$$y = 3 - (x+1)^2$$

A quadratic graph has minimum point (-1, 2). Find an equation for the graph.

A quadratic graph has maximum point (2, 5). Find an equation for the graph.

7. Write each of the following quadratic functions in completed square form:

(i)
$$x^2 + 2x - 3$$

(ii)
$$x^2 - 6x + 1$$

(iii)
$$x^2 + x + 1$$

(iv)
$$-x^2 + 5x$$

(v)
$$2x^2 + 4x + 3$$

(vi)
$$3x^2 + 8x - 2$$

8. Using your answers for each of the quadratic functions in question 7, write down the coordinates of the minimum or maximum point (the vertex) of the graph.

(i)
$$y = x^2 + 2x - 3$$

(ii)
$$y = x^2 - 6x + 1$$

(iv) $y = -x^2 + 5x$

(iii)
$$y = x^2 + x + 1$$

(iv)
$$y = -x^2 + 5x$$

(v)
$$y = 2x^2 + 4x + 3$$

(v)
$$y = 2x^2 + 4x + 3$$
 (vi) $y = 3x^2 + 8x - 2$

Extension

(i) Write $x^2 - 3x + 1$ in the completed square form.

(ii) Hence write down the equation of the line of symmetry and the coordinates of the vertex of the graph $v = x^2 - 3x + 1$.

(iii)Sketch the graph.

Quadratic formula

1. Without solving the equation, state how many solutions there are for each of the following quadratic equations:

(i)
$$3x^2 + 2x + 5 =$$

$$3x^2 + 2x + 5 = 0$$
 (ii) $2x^2 - 3x - 2 = 0$

(iii)
$$5x^2 - 6 = 0$$

$$5x^2 - 6 = 0$$
 (iv) $4x^2 - 8x + 4 = 0$
 $x^2 - 3x + 3 = 0$ (vi) $-5x^2 - 8x - 10$

(v)
$$x^2 - 3x + 3 = 0$$

(vi)
$$-5x^2 - 8x - 10$$

2. Use the quadratic formula to solve these equations. Give your answers in exact form.

(i)
$$x^2 + 4x + 1 = 0$$

(ii)
$$x^2 - 3x - 1 = 0$$

(i)
$$x^2 + 4x + 1 = 0$$
 (ii) $x^2 - 3x - 1 = 0$ (iv) $3x^2 - 4x - 2 = 0$

(iv)
$$3x^2 - 4x - 2 = 0$$

Extension

 A cylinder has height 20 cm and surface area 300 cm². Find the radius of the cylinder, to 3 s.f.

Simultaneous Equations

Solve the following simultaneous equations:

(i)
$$2x + 5y = 11$$

(ii)
$$x + 2y = 6$$

$$2x - y = 5$$

$$4x + 3y = 4$$

(iii)
$$3a-2b=4$$

 $5a+4b=3$

(iv)
$$2p-5q=5$$

 $3p-2q=-9$

(v)
$$5x + 3y = 9$$

 $y = 3x - 4$

(vi)
$$3a+2b=1$$

 $9a-4b=4$

Solve the following simultaneous equations.

$$x + y = 9$$

(ii)
$$xy = 8$$

$$x^2 - 3xy + 2y^2 = 0$$

$$3x - y = 10$$

(iii)
$$y = 4x$$

$$3y^2 - 2xy = 160$$

Extension

- (i) By completing the square, find the coordinates of the vertex of the graph $v = x^2 + x + 1$.
- (ii) By putting the two expressions equal to each other in a single equation, find where the two graphs below cross:

$$y = x^2 + x + 1$$

$$y = 5x - 3$$

(iii)Interpret your result by sketching the graphs.

Inequalities

Solve the following linear inequalities.

(i) 2x+3<10

(ii) $5x+3 \ge 2x-9$

(iii) 3x-1>7-x

(iv) $4x+1 \le 6x-7$

(v) 5x+2>-7

(vi) $3x-11 \le 5+4x$

(vii) $3(2-3x) \ge 5x+1$

(viii) $\frac{1}{3}(7+6x) < 2-x$

- 2. (i) Write $x^2 11x + 24$ in factorised form.
 - (ii) Sketch the graph of $y = x^2 11x + 24$, labelling the values of x where the graph crosses the x-axis.
 - (iii) Use your graph to write down the solution of

$$x^2 - 11x + 24 \ge 0$$

Solve the following quadratic inequalities.

(i) $x^2 - 4x - 12 \le 0$

(iii) $x^2 + 2x - 15 \ge 0$

(ii) $x^2 - 7x + 6 > 0$ (iv) $2x^2 - 5x - 3 \le 0$ (vi) $4x^2 - 4x - 3 > 0$

(v) $3x^2 + 5x + 2 < 0$

Extension

Solve the following quadratic inequalities.

(i) $1-x-2x^2 \ge 0$

(ii) $x^2 + 2x - 1 < 0$

(iii) $x^2 \ge 3x + 10$ (iv) x(x+3) > x+8